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BENDING STRENGTH OF UNSYMMETRICAL PLATE GIRDERS

by

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and

Alexis Ostapenko[2]

ABSTRACT

An unsymmetrical plate girder is defined as a girder whose centroidal axis is not at the mid-depth of the web because of the unequal areas of the flanges. The ultimate static strength of such girders subjected to pure bending is determined theoretically. The ultimate moment is assumed to consist of two contributions: the girder moment before buckling of the web and the girder moment controlled by the flange strength remaining after buckling of the web. In the buckling analysis, the web panel is assumed to be fixed at the flanges and pinned at the stiffeners. After web buckling, the compression portion of the web is considered as being replaced with an effective plate strip which is a part of the compression flange. A girder panel can reach its ultimate capacity before or after buckling of the web, by the failure of the compression flange or by the yielding of the tension flange. The method is applicable to symmetrical and unsymmetrical plate girders with a homogeneous or hybrid cross section. It compares well with the available test results and thus provides a reliable means of computing the ultimate strength of transversely stiffened plate girders.

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1. INTRODUCTION

Plate girders have been traditionally designed using the buckling stress of the web plate as the controlling criterion. The post-buckling strength of the web was recognized indirectly by employing a factor of safety against the buckling of the web lower than, for example, against the yielding of the flanges. Only since the early sixties, the ultimate strength has been introduced directly as a design criterion.[1,13]

The behavior of a web plate panel subjected to an increasing girder moment can be briefly described by means of Fig. 1. Although, due to initial web deflections, the true sudden web buckling seldom takes place, the deflections are usually small up to approximately the theoretical buckling stress intensity σ_{cp} and the stress remains essentially linearly distributed as shown in Fig. 1a.[3,4,6,7,9,12] As the girder moment gradually goes above the buckling value, the web stresses close to the compression edge increase beyond the plate buckling stress, as shown in Fig. 1b. At some distance from the compression edge, where the bulge forms, the stresses remain of approximately the same intensity or even decrease while the lateral deflection of the web progresses continuously. On the other hand, the stresses in the tension part of the web, although growing in intensity, remain linearly distributed. An exact analysis of the web stresses is still too complex, and some idealizations have been proposed, such as the one shown in Fig. 1c, where the post-buckling contribution of the compression portion of the web is handled by the effective width concept (t.[7] The ultimate moment capacity is reached when the tension portion of the web and the tension flange yield or when the compression flange yields or buckles.

Basler and Thurlimann were the first to present (in 1961) an analysis of the ultimate strength of plate girders under bending, on the basis of an extensive experimental program.[2,3,4] Consideration was given to the buckling and post-buckling contributions of the web plate. In 1965, Konishi modified their bending strength formula by introducing a specific magnitude of residual stresses for each grade of steel.[9] Then in 1968, Lew employed a similar technique to obtain an approach for hybrid girders.[10] In 1968, Fujii presented his theory based on the assumption that the ultimate bending strength of a girder panel is reached when the sum of the flange stress at the web buckling moment and the flange stress due to the post-buckling strength of the web reaches the yield stress of the flange.[7]

In all the above studies, the web panel was assumed to be pinned at all edges and only symmetrical plate girders were considered, that is, the centroidal axis was assumed to be at the mid-depth of the web. However, it can be readily recognized that the web plate must be restrained at the flanges and stiffeners and thus its buckling strength should be greater than if pinned edges were assumed. Also, many plate girders in practical construction are unsymmetrical, such as, for example, the composite and orthotropic deck sections shown in Fig. 2. Although reference 13 suggests that unsymmetrical sections should be treated as symmetrical ones with the depth equal to the double of the compression portion of the web, the problem of unsymmetrical plate girders can be hardly considered as solved.

The purpose of the study presented here was to formulate a method which would give a better consideration of the true behavior of plate girders than methods proposed previously. When compared with the available test results, the method developed was found to give a reliable estimate of the ultimate bending strength for homogeneous and hybrid girders with symmetrical or unsymmetrical sections.

2. ANALYSIS

The proposed approach gives a complete coverage of the possibilities of yielding or buckling of the compression flange or plastification of the whole cross section including the web.

The ultimate strength of transversely stiffened girder panels under pure bending, as shown in Fig. 1c, is assumed to be given by the sum of (a) the web buckling moment M_b, and (b) the post-buckling moment M_t which is required to produce failure of the compression flange column by buckling or yielding.

$$M_u = M_b + M_t \tag{1}$$

Buckling strength.- The web buckling capacity of a girder panel is obtained from the ordinary beam theory formula

$$M_b = \sigma_{cp} \frac{I}{y_c} \tag{2}$$

where I is the moment of inertia of the cross section about its horizontal centroidal axis, y_c is the distance from the centroidal axis to the compression flange-web junction, and σ_{cp} is the compressive buckling stress for bending.

$$\sigma_{cp} = k_b \frac{\pi^2 E}{12\ (1-\nu^2)} \left(\frac{t}{b}\right)^2 \tag{3}$$

The bending buckling coefficient k_b is a function of the stress distribution and the web plate boundary conditions. A study showed that the effect of the elastic restraint furnished by the edge members to the web plate can be simplified by assuming that the web is fixed at the flanges and pinned at the stiffeners as shown in Fig. 1a.[12] Then, k_b can be given by

$$k_b = 13.54 - 15.64\ C + 13.32\ C^2 + 3.38\ C^3 \tag{4}$$

Where $C = \sigma_2/\sigma_1$ is the ratio of σ_2 the maximum tensile stress (or minimum compressive stress) to the maximum compressive stress σ_1 as shown in Fig.3. C is negative when the bottom fiber is in tension. In this equation, k_b is conservative since the effect of the aspect ratio α is not included by assuming a minimum value of k_b for α equal to infinity. This may be too conservative primarily for α less than 0.5 which is seldom of practical significance. Equation 4 is shown by curve (1) in Fig 4. A lower curve (2) in the figure is for a plate pinned at its four edges as has been used by other investigators.[3,7]

Two limitations are imposed on equation 2 to account for the influence of the strength of the boundary members. The first is that the web buckling stress σ_{cp} has to be less than or equal to the buckling stress of the compression flange column σ_{cf}. Otherwise the buckling of the compression flange will take place prior to the buckling of the web. Therefore, when $\sigma_{cp} \geq \sigma_{cf}$, the ultimate strength of the panel is

$$M_u = \sigma_{cf} \frac{I}{y_c} \tag{5}$$

The second limitation is that the tension stress in the extreme fiber must be less than the tension flange yield stress σ_{yt}. If this condition is violated, the yielding in the tension flange takes place prior to the buckling of the web. Once the tension flange reaches yield stress, yielding may start penetrating into the web. At the same time, the stress in the compression flange will increase until it reaches the flange column buckling stress. If the compression flange is continuously supported, it will reach the yield stress. Then the section will be fully plastified, and the moment capacity will be the plastic moment M_p (see Appendix II). Therefore, when * $|C\ \sigma_{cp}| \geq |\sigma_{yt}|$, the ultimate strength of the panel is taken as

$$M_u = M_p \tag{6}$$

Post-Buckling strength. - An additional moment required to produce buckling or yielding of the compression flange or yielding of the tension portion after buckling of the web plate is called the post-buckling moment M_t. Steps in developing the pertinent equations for M_t are facilitated by using the following non-dimensionalized parameters

$$\rho_1 = \frac{A_{fc}}{A_w} \qquad \rho_5 = \frac{\sigma_{yt}}{\sigma_{yw}}$$

$$\rho_2 = \frac{A_{ft}}{A_w} \qquad \rho_8 = \frac{-C\sigma_{cf}}{\sigma_{yt}} \tag{7a}$$

$$\rho_4 = \frac{\sigma_{yc}}{\sigma_{yw}}$$

$$\eta_c = \frac{y_c}{b} \qquad \eta_f = \frac{\bar{d}_t}{b}$$

$$\eta_t = \frac{y_t}{b} \qquad \eta_g = \frac{y_o}{b} \tag{7b}$$

$$\eta_d = \frac{\zeta t}{b} \qquad \eta_h = \frac{2c_c}{t}$$

$$\eta_e = \frac{\bar{d}_c}{b}$$

where A_{fc}, σ_{yc}; A_{ft}, σ_{yt}, and A_w, σ_{yw} are, respectively, the cross sectional area and yield stress of the compression flange, tension flange, and the web plate. The other symbols are shown in Fig. 5a and Fig. 12a and described in Appendix IV (Notation).

The location of the centroidal (neutral) axis from the tension flange-web junction of the effective post-buckling section shown in Fig. 5a is defined by η_t in terms of the non-dimensionalized parameters ρ's and η's

$$\eta_t = \sqrt{(\rho_1 + \rho_2 + \eta_d)^2 + 2\,(\rho_1 + \rho_1\,\eta_e + \eta_d - \frac{1}{2}\,\eta_d^2 - \rho_2\,\eta_f)} - (\rho_1 + \rho_2 + \eta_d) \tag{8a}$$

The location of the centroidal axis from the compression flange-web junction is then

$$\eta_c = 1 - \eta_t \tag{8b}$$

Equilibrium of the horizontal forces of Fig. 5b gives the following stress relationship:

$$\sigma_2' = -\left(\frac{\rho_1 + \eta_d}{\rho_2 + 0.5\,\eta_t}\right)\sigma_1' \tag{9}$$

where σ_1' and σ_2' are, respectively, the compression and tension flange stresses developed after web buckling. When instability of the compression flange is the controlling factor, the post-buckling strength of the girder is evaluated by using

$$\sigma_1' = \sigma_{cf} - \sigma_{cp} \tag{10}$$

M_t is obtained by summing about the neutral axis the moments contributed by the compression flange, the effective compression portion of the web, the tension portion of the web, and the tension flange. Then

$$M_t = \psi_c\, M_{fc} \tag{11}$$

where

$$M_{fc} = \sigma_1'\, A_{fc}\, b \tag{12a}$$

and

$$\psi_c = \eta_c + \eta_e + \frac{\eta_d}{\rho_1}\left(\eta_c - 0.5\,\eta_d\right)$$

$$+ \frac{\rho_1 + \eta_d}{\rho_2 + 0.5\,\eta_t}\left[\frac{1}{3}\frac{\eta_t^2}{\rho_1} + \frac{\rho_2}{\rho_1}\left(\eta_t + \eta_f\right)\right] \tag{12b}$$

Modes of Failure of Compression Flange. - The stress distribution shown in Fig. 5b and the corresponding ultimate moment are controlled by the stability of the compression flange or yielding of the tension flange. In order to deal with the instability modes of failure of the compression flange, the flange is treated as an isolated column with three modes of buckling: lateral, torsional, and vertical. These three modes are shown in Fig. 6, and will be discussed individually next.

Lateral Buckling. - By considering the compression flange acting with the effective depth of the web as a column subjected to compression at its ends (Fig. 7), a simple estimate of the lateral buckling stress is obtained from Ref. 8:

$$\left(\frac{\sigma_{cf}}{\sigma_{yc}}\right)_{\ell} = \frac{1}{\lambda_{\ell}^{2}} \tag{13a}$$

where

$$\lambda_{\ell} = \frac{\ell}{r}\sqrt{\frac{\varepsilon_y}{\pi^2}}$$

and

$$r = \sqrt{I_f / \left(A_{fc} + \zeta t \cdot t\right)}$$

ℓ is laterally unsupported length, ε_y is the flange yield strain, r is the radius of gyration of the compression flange column in the lateral direction, I_f and A_{fc} are, respectively, the moment of inertia and the area of the compression flange, and ζt is the effective web depth (see Appendix I).

Equation 13a is applicable only in the elastic range. It has been suggested that the CRC basic column formula be used in the inelastic range, with the compressive residual stress taken to be $\sigma_{yc}/2$ at $\lambda_\ell = \sqrt{2}$.(8) Thus for $0 \leq \lambda_\ell \leq \sqrt{2}$

$$\left(\frac{\sigma_{cf}}{\sigma_{yc}}\right)_\ell = 1 - \frac{\lambda_\ell^2}{4} \tag{13b}$$

Equations 13a and 13b giving the lateral buckling stress of the compression flange column of a plate girder subjected to pure bending, are plotted in Fig. 8.

Torsional Buckling.- This is essentially the local buckling of the flange plate and is analyzed by the usual methods. By considering the compression flange as a long plate hinged at the flange-web junction and subjected to edge compression at its ends, Fig. 9, a torsional buckling equation in the elastic range is obtained,

for $\lambda_t > \sqrt{2}$

$$\left(\frac{\sigma_{cf}}{\sigma_{yc}}\right)_t = \frac{1}{\lambda_t^2} \tag{14a}$$

where

$$\lambda_t = \frac{c_c}{d_c}\sqrt{\frac{12\,(1-\nu^2)\,\varepsilon_y}{\pi^2\,k_t}}$$

c_c and d_c are, respectively, the half-width and thickness of the compression flange, ε_y is the flange yield strain, k_t is the torsional buckling coefficient of the flange plate. Conservatively assuming that the flange is pinned at both ends and that the web plate gives no rotational restraint and the flange buckles as if each half of the flange was pinned at the web, coefficient k_t is 0.425.[8]

In the inelastic range, it is assumed that the magnitude of the compressive residual stress is $\sigma_{yc}/2$ at $\lambda_t = \sqrt{2}$, that strain-hardening commences at $\lambda_t = 0.45$, and that a reasonably smooth transition curve is tangent to the buckling curve at $\lambda_t = 2$ and to the yield strength curve at $\lambda_t = 0.45$.[8] Then

for $$0.45 \leq \lambda_t \leq \sqrt{2}$$

$$\left(\frac{\sigma_{cf}}{\sigma_{yc}}\right)_t = 1 - 0.53\left(\lambda_t - 0.45\right)^{1.36} \tag{14b}$$

Equations 14a and 14b are plotted in Fig. 10. In order to eliminate torsional buckling as a primary cause of failure, the following design criterion has been suggested [3,8]:

$$\frac{2\,c_c}{d_c} \leq 12 + \frac{\ell}{2\,c_c} \tag{15}$$

Alternatively, the width-thickness ratio requirements of Art. 1.9 of the AISC Specification may be used. [1]

Vertical Buckling. - Vertical movement of the compression flange has been studied by several investigators.[2,5,10] The conclusions drawn in Reference 5 and substantiated by others are that before vertical buckling of the compression flange can take place, both of the following conditions must be fulfilled: (i) the web plate must be slender enough to allow the development of large lateral web deflections in order that the resistance to vertical buckling of the compression flange becomes negligible, and (ii) the compression flange must be fully yielded so that its bending rigidity also becomes negligible. However, according to Equations 13 and 14, the critical stresses at which a compression flange can no longer resist lateral buckling or torsional buckling will be reached prior to the yielding of the compression flange (condition ii). It appears, therefore, that vertical buckling of the compression flange can only take place after the ultimate bending moment has been attained and the girder subjected to additional deformations.

Ultimate Moment. - The final form of the static ultimate bending strength formula is obtained when Eqs. 2 and 11 are inserted into Eq. 1.

$$M_u = \sigma_{cp} \frac{I}{y_c} + \psi_c M_{fc} \tag{16}$$

Equation 16 is applicable as long as the ultimate moment is not higher than the value given by Equation 5 or 6.

Mode of Failure as a Function of Location of Centroidal Axis. - Figure 11 illustrates how the mode of failure depends on the location of the centroidal axis y_t/b and on the web slenderness ratio β.* Above the solid line is the region in which the girder panel fails by the yielding of the tension portion of the section. Below it, is the region where the ultimate capacity of the panel is limited by the buckling of the compression flange. The solid line is almost horizontal, thus indicating that the mode of failure is not overly dependent on the slenderness ratio β.

The dashed line separates the left region in which either type of failure may take place before the buckling of the web and the right region in which the failure occurs after the web plate buckles. Noteworthy is the fact that as the portion of the section in tension increases the web buckling is inhibited for higher and higher values of β. This means, for example, that a composite beam in the positive moment region, in which most of the web is in tension, may be designed with a higher β and a wider spacing of stiffeners than a non-composite girder.

3. COMPARISON WITH TEST RESULTS

The ultimate bending strength theory is compared with the available twenty four experiments carried out on symmetrical, unsymmetrical and hybrid girders. The girders and the tests are described in detail in References 4,5,6,9 and 10. Table 1 summarizes the dimensions of the test

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* The plot was made for the following particular combination of girder parameters: $\ell/r \approx 50$, $\rho_1 = A_{fc}/A_w = 1.2$, $\sigma_{yw} = \sigma_{yc} = \sigma_{yt} = 36$ ksi, and, thus, is not valid in general.

panels and the material properties of the girder components. The experimental ultimate load P_{ex}, the predicted load calculated by the proposed approach P_u and the ratio of P_{ex}/P_u are tabulated in columns 12, 13 and 14 of the table. The experimental results compared with the values computed by the methods of Reference 3 and 10 are also listed in columns 15 and 16 for reference. The mode of compression flange buckling observed for each test is indicated in column 17 by L (lateral), T (torsional), or V (vertical).

Ten tests on homogeneous girders which failed by lateral buckling of the compression flange are in good agreement with the predicted loads evaluated by either the proposed approach or the Basler's approach.[3] However, two tests on unsymmetrical girders, UG1.2 and UG 2.3, indicate that the proposed approach gives a somewhat better correlation with tests than Basler's method modified according to Reference 13. For the four girder panels, which showed vertical buckling of the compression flange, the proposed approach gives a conservative estimate with an average error of 5% and a maximum of 10%.

The average deviation of the ten tests with torsional buckling of the compression flange is 7%. However, four out of the ten tests deviated more than 10%, and except for girder B-5 of Ref. 10, they were on the conservative side. The reasons for the conservative deviation may be the following: (a) the magnitude of the compressive residual stress taken as $\sigma_{yc}/2$ at $\lambda_t = \sqrt{2}$ is too conservative for steels with yield stress greater than 33 ksi,[10,11] and (b) the assumptions made for the torsional buckling coefficient k_t are conservative since the rotational restraint by the web is not considered.

The proposed approach, thus, provides a reliable estimate of the ultimate bending strength of homogeneous and hybrid girders with symmetrical or unsymmetrical cross section. However, its advantages over other methods are not only its generality and greater accuracy, but the fact that the analytical model of the panel behavior, on which the method is based, is very suitable for considering interaction when shear is applied to the panel in addition to the moment. With some modifications the model can be also extended to panels with longitudinal stiffeners. These topics are discussed in other reports.

4. SUMMARY AND RECOMMENDATIONS FOR FUTURE WORK

A general method for determining the ultimate bending strength of plate girders was developed. It is applicable to homogeneous and hybrid, unsymmetrical girders (a symmetrical girder is a particular case of an unsymmetrical girder). The main features of the method are:

1) The ultimate moment of girders with slender webs is given as a sum of the buckling and post-buckling moments.
2) The limiting criterion may be the compression flange failure by buckling or yielding, or the tension flange yielding depending on the location of the horizontal centroidal axis relative to the girder depth and on the web slenderness ratio.
3) In buckling computation, the web is assumed to be fixed at the flanges and pinned at the stiffeners.
4) The method gives close correlation with the available test results, yet it does not utilize experimental data for establishing any coefficients as do other methods.

5) The analytical model of the method is suitable for extending the method to girders under combined loads and to longitudinally stiffened plate girders.

The following items are suggested for future research to make this method even more accurate:

a) Refinement of the buckling coefficient k_b to make it dependent on the aspect ratio α. This should improve the accuracy for α less than about 0.5.

b) The effective width ζt of the compressed web in the post-buckling range.

c) Vertical buckling of the compression flange, in particular, for high β and σ_{yw} much less than σ_{yc}.

d) Torsional (local) buckling of the compression flange.

e) Effective length and residual stresses of the compression flange column.

f) Lateral stability of the tension flange of very deep girders.

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6. APPENDIX I - EFFECTIVE WIDTH OF WEB PLATE

Little information is available on the effective width of a rectangular plate subjected to in-plane bending. It has been suggested that the effective web depth of 30t is applicable to a symmetrical girder with the web yield stress of 33 ksi when the web slenderness ratio is $\beta = 360$.[3] On the other hand, when the web buckling stress coincides with the yield stress, only a small portion of the web can contribute to act with the compression flange as a column.

By assuming that the stress distribution on the cross section of a girder panel in the post-buckling range is as shown in Fig. 5b, the effective width coefficient may be approximately taken as

$$\zeta = \frac{1.33}{\sqrt{\varepsilon_{yw}}} \left(1 - \frac{\sigma_{cp}}{\sigma_{yw}}\right) \tag{17}$$

The coefficient ζ is equal to 30 when $\beta = 360$ for symmetrical girders and to zero when the bending buckling stress σ_{cp} coincides with the web yield stress of 33 ksi.

7. APPENDIX II.- PLASTIC MOMENT

Plastic moment M_p of an unsymmetrical plate girder section enters into Eq. 6. It is evaluated as follows:

Neutral Axis in the Web. - When the summation of the normal forces of the cross section shown in Fig. 12 is set equal to zero, the nondimensionalized parameter η_g to define the location of neutral axis is obtained.

$$\eta_g = \frac{y_o}{b} = \frac{1}{2}\left(\rho_2\,\rho_5 + 1 - \rho_1\,\rho_4\right)$$

The plastic moment capacity of the cross section, is evaluated by taking the sum of the moments contributed by the compression flange, the web plate, and the tension flange about the neutral axis.

$$M_p = \psi_t\,M_{ft} \tag{18}$$

where

$$M_{ft} = A_{ft}\,\sigma_{yt}\,b$$

and

$$\psi_t = 1 + \eta_f - \eta_g + \frac{1}{\rho_2\,\rho_5}\left[\rho_1\,\rho_4\left(\eta_e + \eta_g\right) + \eta_g^2 - \eta_g + 0.5\right] \tag{19}$$

Neutral Axis in the Compression Flange. - The location of the neutral axis from the compression flange-web junction in the section of Fig. 13 is expressed by the nondimensionalized parameter

$$\eta_g = \frac{1}{2}\left[\rho_1 \rho_4 - \left(1 + \rho_2 \rho_5\right)\right]\frac{1}{\eta_h \rho_4}$$

The moment capacity of this cross section is given then by

$$M_p = \psi_t' M_{ft} \tag{20}$$

where

$$\psi_t' = 1 + \eta_f + \eta_g + \frac{1}{\rho_1 \rho_5}\left[\rho_1 \rho_4 \left(\eta_e - \eta_g\right) - \rho_4 \eta_g^2 \eta_h + \eta_g + 0.5\right] \tag{21}$$

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9. APPENDIX IV. - NOTATION

1. Lower Case Letters

a	Panel width or distance between transverse stiffeners.
b	Panel depth or distance between flanges.
c_c	Half width of a compression flange.
c_t	Half width of a tension flange.
d_c	Compression flange thickness.
$\bar{d}_c$	Distance from the compression flange-web junction to the centroid of the compression flange.
d_t	Tension flange thickness.
$\bar{d}_t$	Distance from the tension flange-web junction to the centroid of the tension flange.
k_b	Plate buckling coefficient under pure bending
k_t	Plate buckling coefficient for torsional buckling.
ℓ	Lateral unsupported length of the compression flange.
r	Radius of gyration of compression flange column.
t	Web thickness.
y_c	Distance from the centroidal axis to the extreme compressive fiber of the web for the effective section before or after web buckling.
y_t	Distance from the centroidal axis to the extreme tensile fiber of the web for the effective section before or after web buckling.
y_o	Distance from the neutral axis to the compression flange-web junction for a fully yielded cross section.

2. Capital Letters

A_{fc}	Area of compression flange.
A_{ft}	Area of tension flange.

2. Capital Letters Cont'd

A_w	Web area.
C	The ratio of the maximum tensile stress (or minimum compressive stress) to the maximum compressive stress of the web plate (for positive moment, C is the ratio of the bottom fiber stress to the top fiber stress). Note that C is negative when the bottom fiber is in tension.
E	Modulus of elasticity, 29,600 ksi.
I	Moment of inertia of the girder cross section about the horizontal axis.
I_f	Moment of inertia of the compression flange about the vertical axis.
M_b	Moment at web buckling load.
M_{fc}	Moment contributed by the compression flange.
M_{ft}	Moment contributed by the tension flange.
M_p	Plastic moment of a fully yielded cross section.
M_t	Additional moment after web buckling.
P_{ex}	Experimental ultimate load.
P_u	Theoretical ultimate load.

3. Greek Letters

α	Panel aspect ratio = a/b.
β	Web slenderness ratio = b/t.
ε	Strain.
$\left.\begin{matrix}\eta_c, \eta_t, \\ \eta_d, \eta_e, \\ \eta_f, \eta_g, \\ \eta_h\end{matrix}\right\}$	Non-dimensionalized parameters, see Eq. 7b for definitions.

λ	Slenderness parameter used in column curves, Figs. 8 and 10.
ν	Poisson's ratio, 0.3 for steel.
ρ_1, ρ_2, ρ_4, ρ_5, ρ_8	Non-dimensionalized parameters, see Eq. 7a for definitions.
σ_{cf}	Buckling stress of compression flange.
σ_{cp}	Buckling stress of web plate.
σ_1	Compressive stress of the extreme fiber of the web before web buckling.
σ_1'	Additional compressive stress of the extreme fiber of the web after web buckling.
σ_2	Tensile stress of the extreme fiber of the web before web buckling.
σ_2'	Additional tensile stress of the extreme fiber of the web after web buckling.
σ_{yc}	Yield stress of compression flange.
σ_{yt}	Yield stress of tension flange.
σ_{yw}	Yield stress of web.
ζ	Coefficient of the effective width of the web plate.
ψ_c	Coefficient of post-buckling moment.
ψ_t	Coefficient of plastic moment.

10. TABLES AND FIGURES

Table 1. Comparison of Theory with Test Results

Source	Test No.	α	β	ℓ	Web $b \times t$	Web σ_{yw}	Compr. Flange $2c_c \times d_c$	Compr. Flange σ_{yc}	Tens. Flange $2c_t \times d_t$	Tens. Flange σ_{yt}	P_{ex}	P_u	$\frac{P_{ex}}{P_u}$	$\frac{P_{ex}}{P_{uB}}$	$\frac{P_{ex}}{P_{uL}}$	Mode of Failure
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
				in.	in. x in.	Ksi	in. x in.	Ksi	in. x in.	Ksi	Kips	Kips				
Ref.4	G1-T1	1.5	185	100	50.0 x .270	33.0	20.56 x .427	35.4	12.25 x .760	38.5	81	72.7	1.11	1.11	--	T
	G2-T1	"	"	"	"	35.3	12.19 x .769	38.6	12.19 x .774	37.6	135	142	.95	.96	--	L
	G2-T2	0.75	"	50	"	"	"	"	"	"	144	144	1.00	.99	--	L
	G3-T1	1.5	"	100	"	33.7	ø8.62 x .328	35.5	12.19 x .770	38.1	130	129	1.01	1.03	--	L
	G3-T2	0.75	"	50	"	"	"	"	"	"	136	132	1.03	1.05	--	L
	G4-T1	1.5	388	100	50.0 x .129	43.4	12.16 x .774	37.6	12.19 x .765	37.0	118	118	1.00	1.00	--	L
	G4-T2	0.75	"	50	"	"	"	"	"	"	125	120	1.04	1.03	--	V
	G5-T1	1.5	"	100	"	45.7	ø8.62 x .328	35.5	12.25 x .767	37.0	110	110.5	1.00	1.04	--	L
	G5-T2	0.75	"	50	"	"	"	"	"	"	124	114	1.08	1.14	--	L
Ref.5	LB 1	1.0	444	125	55.0 x .124	33.3	12.01 x .754	37.6	12.01 x .754	37.6	156.5	152	1.03	1.01	--	L
Ref.6	UG1.2	0.8	295	114	36.0 x .120	44.4	8.0 x .625	34.2	8.0 x .625 10.5 x .750	34.2	78	76.0	1.02	1.06	--	L
	UG2.3	1.2	"	138	36.0 x .122	43.2	"	36.7	" "	36.7	63	63.5	1.00	1.09	--	L
	A-1	1.0	298	36	36.0 x .121	33.2	8.04 x .526	104.7	8.04 x .526	104.7	116	111	1.04	--	1.02	V
	A-2	"	140	"	36.0 x .257	36.4	"	"	"	"	128	128.5	1.00	--	1.03	T
	A-3	"	95	"	36.0 x .380	41.5	7.95 x .527	"	7.95 x .527	"	139	141	.99	--	1.06	T
	B-1	1.5	305	54	36.0 x .118	33.9	7.98 x .502	107.6	7.98 x .502	107.6	129	125	1.03	--	1.02	V
Ref.10	B-2	"	277	"	36.0 x .130	34.6	8.01 x .502	"	8.01 x .502	"	140	127	1.10	--	.98	V

(Continued)

Table 1. Comparison of Theory with Test Results (Continuation)

Source	Test No.	α	β	ℓ	$b \times t$	σ_{yw}	Compr. Flange $2c_c \times d_c$	Compr. Flange σ_{yc}	Tens. Flange $2c_t \times d_t$	Tens. Flange σ_{yt}	P_{ex}	P_u	$\frac{P_{ex}}{P_u}$	$\frac{P_{ex}}{P_{uB}}$	$\frac{P_{ex}}{P_{uL}}$	Mode of Failure
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
				in.	in. x in.	Ksi	in. x in.	Ksi	in. x in.	Ksi	Kips	Kips				
Ref.10	B-3	1.5	188	54	36.0 x .191	38.3	8.03 x .502	107.6	8.03 x .502	107.6	148	131	1.13	--	1.04	T
	B-4	"	146	"	36.0 x .247	35.8	8.05 x .259	113.7	8.05 x .259	113.7	55	53	1.04	--	1.18	T
	B-5	"	"	"	"	"	8.03 x .370	108.7	8.03 x .370	108.7	80	93	.86	--	.88	T
	C-4	1.0	147	78	36.0 x .245	41.6	8.05 x .520	105.0	8.05 x .520	105.0	182	194	.94	--	.91	T
	C-5	"	"	"	"	"	8.04 x .519	"	8.04 x .519	"	184	193	.95	--	.93	T
				mm	mm x mm	kg/mm^2	mm x mm	kg/mm^2	mm x mm	kg/mm^2	Ton	Ton				
Ref. 9	A	1.0	267	1200	1200 x 4.5	46.3	240 x 12	28.0	240 x 12	28.0	46.5	44.4	1.05	1.03		T
	C	"	200	1200	1200 x 6.0	52.5	240 x 12	50.0	240 x 12	50.0	96.0	83.2	1.15	1.13		T

Note: ϕ: Pipe flange, 8.62 in. diameter, 0.328 in. thick.

P_u: Predicted Load calculated by using the proposed approach.

P_{uB}: Predicted Load calculated by using Basler-Thurlimann's approach.

P_{uL}: Predicted Load calculated by using Lew's approach.

Observed Mode of Failure
L = Lateral Buckling
T = Torsional Buckling
V = Vertical Buckling

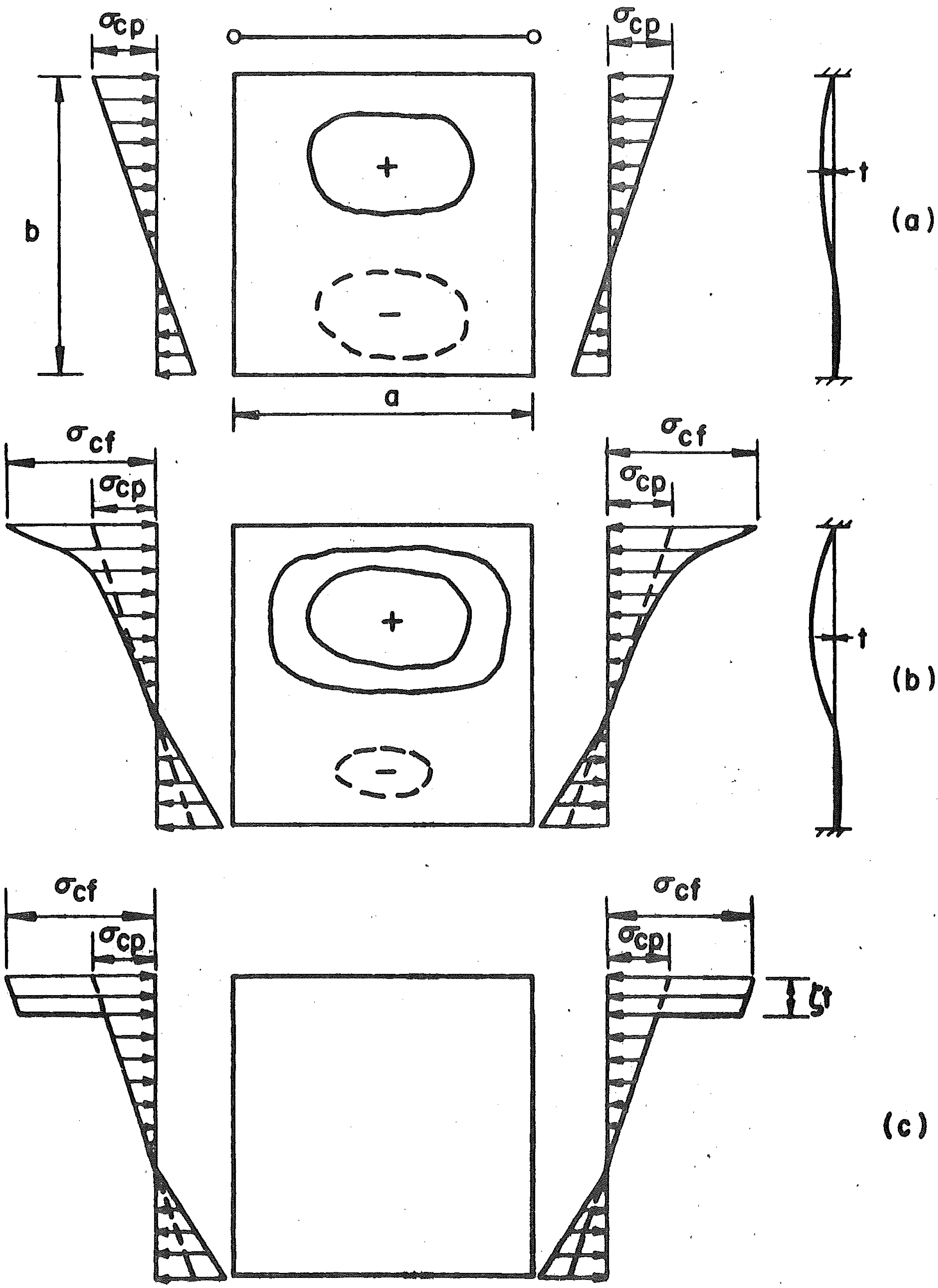

Fig. 1 Behavior of Web Plate Subjected to In-Plane Bending

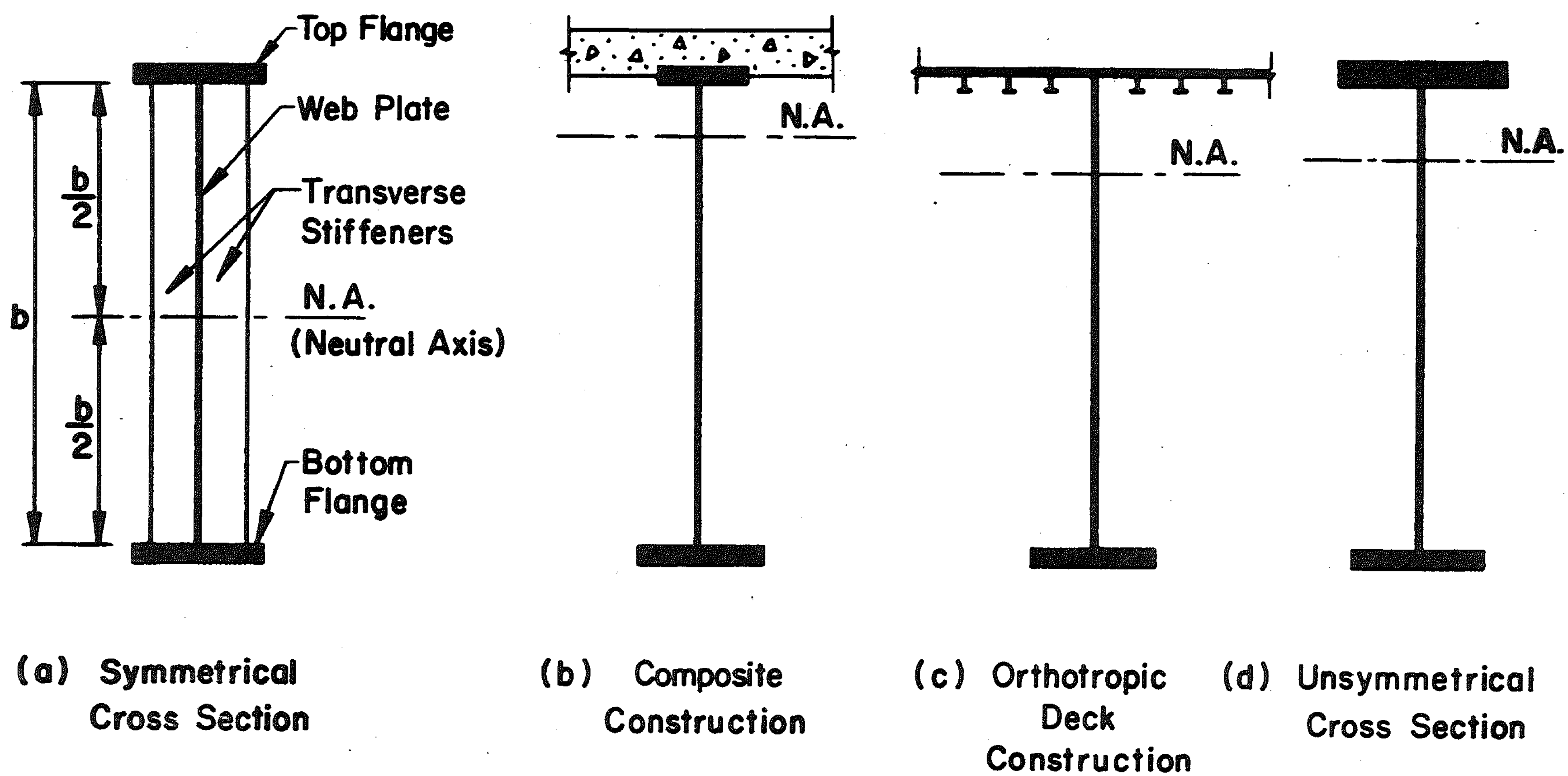

Fig. 2 Symmetrical and Unsymmetrical Plate Girders

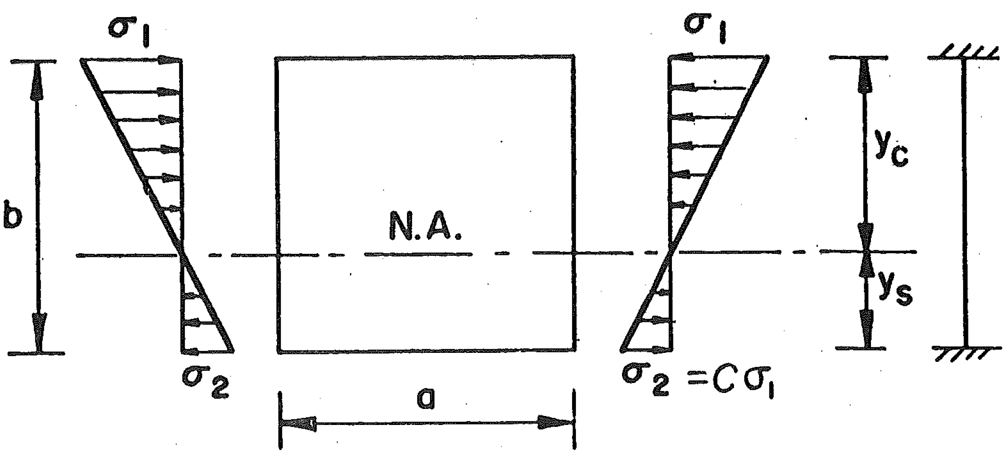

Fig. 3 Stress in Web Before Buckling

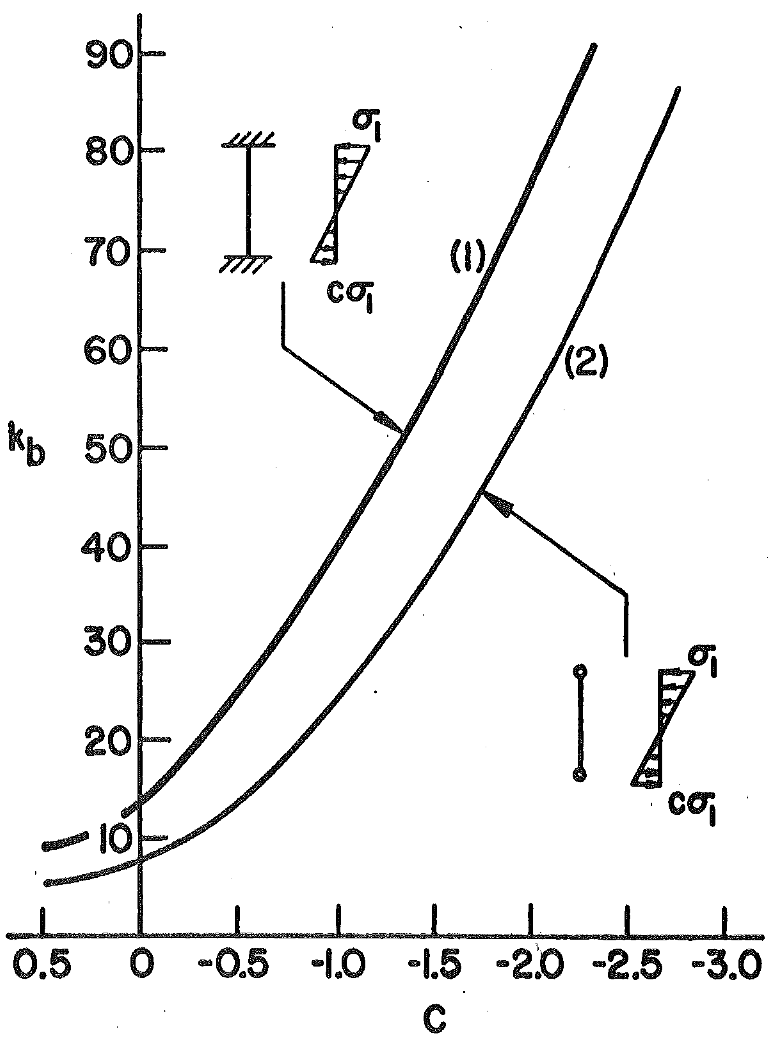

Fig. 4 Bending Buckling Coefficient k_b vs. Stress Ratio C

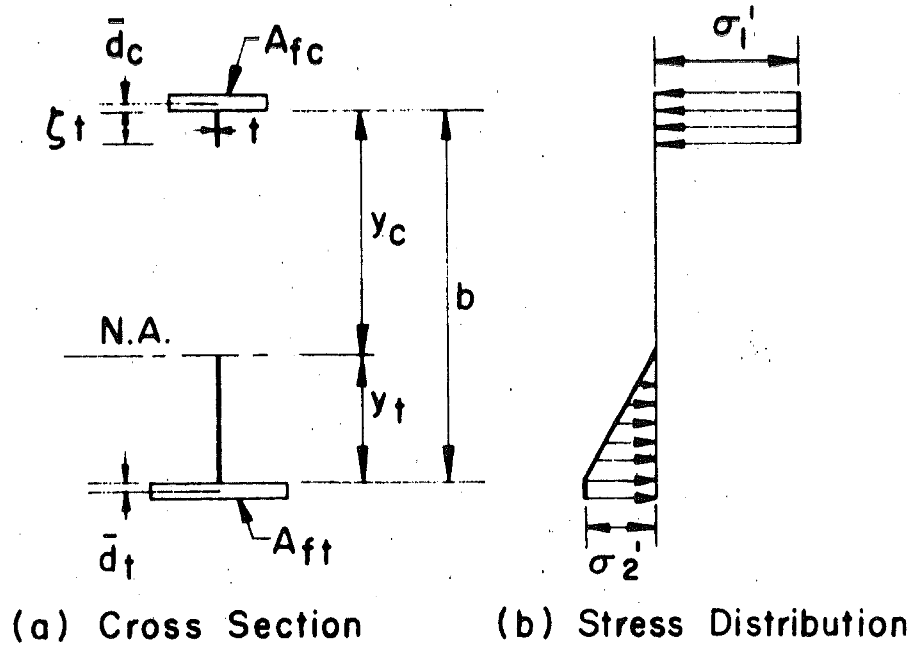

Fig. 5 Post-Buckling Strength Model

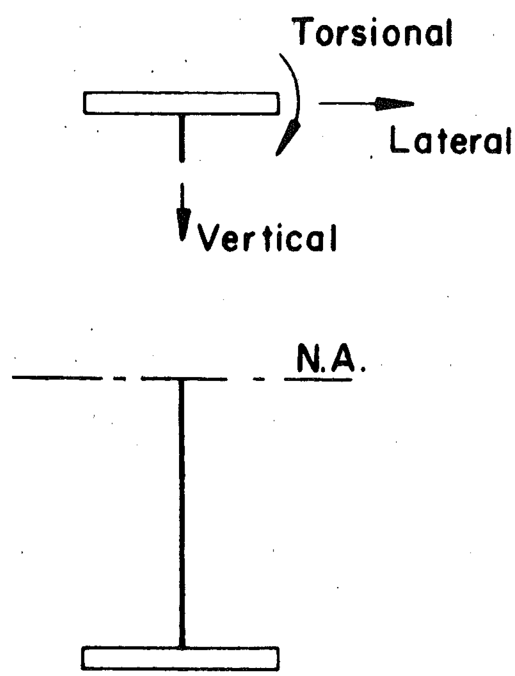

Fig. 6 Buckling Modes of Compression Flange Column

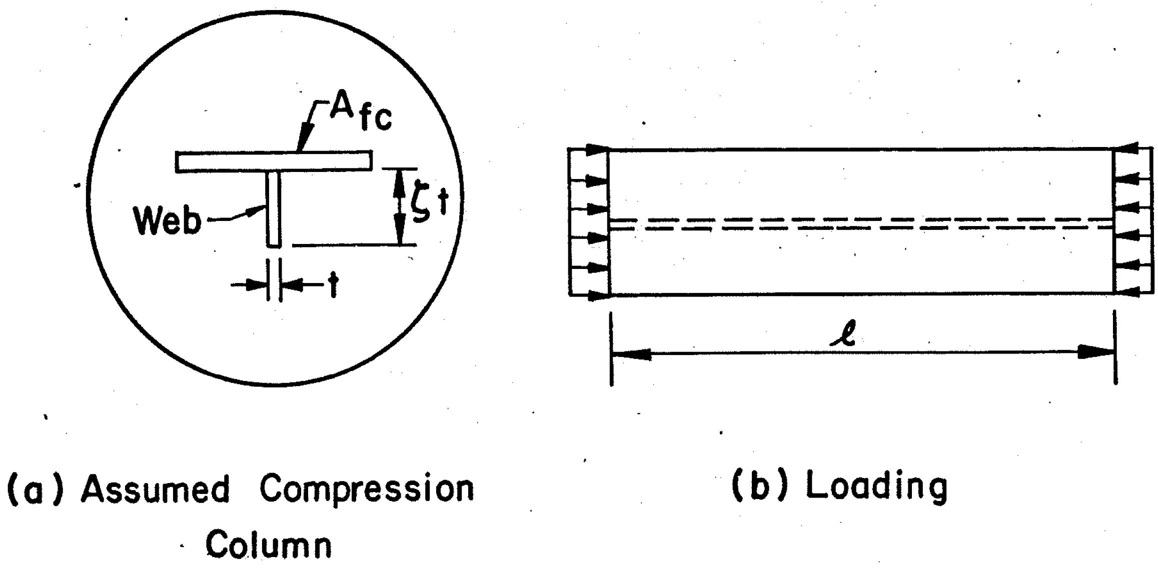

Fig. 7 Lateral Buckling of Compression Flange

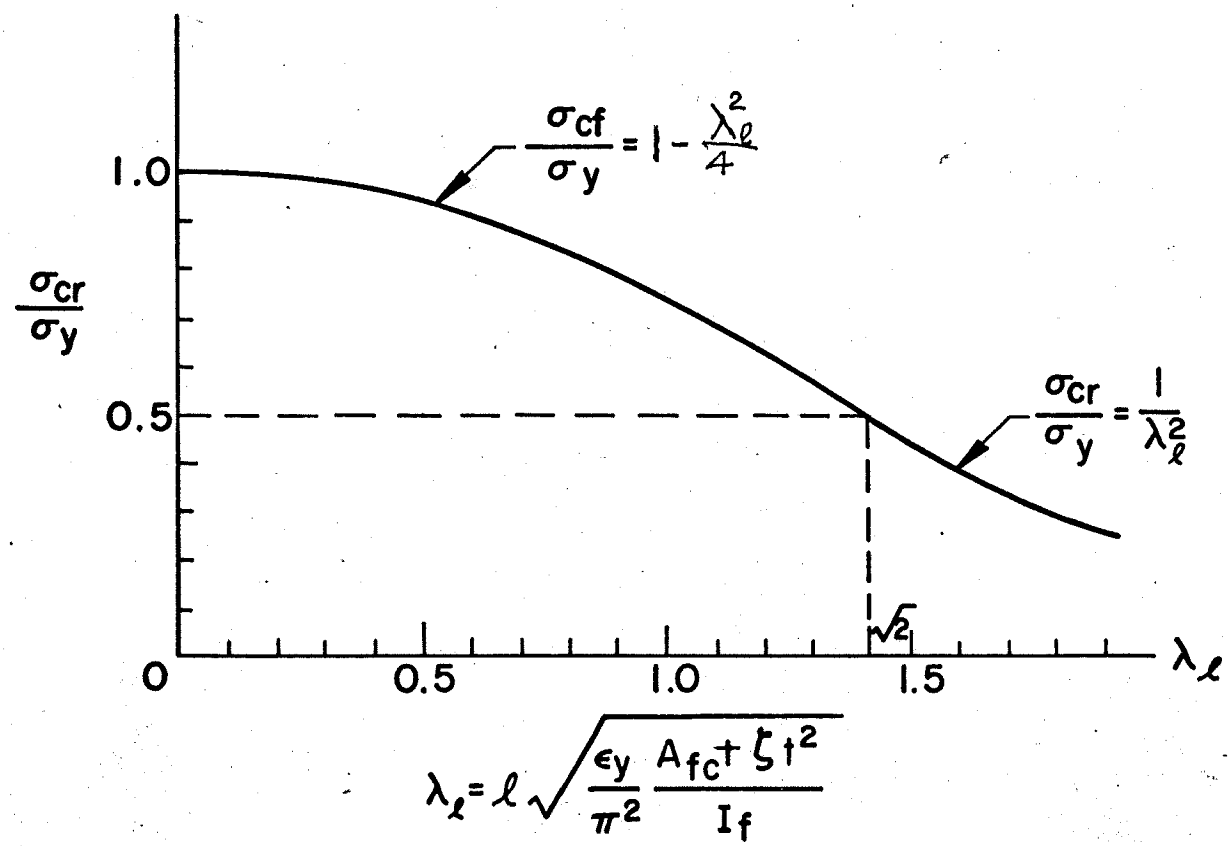

Fig. 8 Lateral Buckling Curve

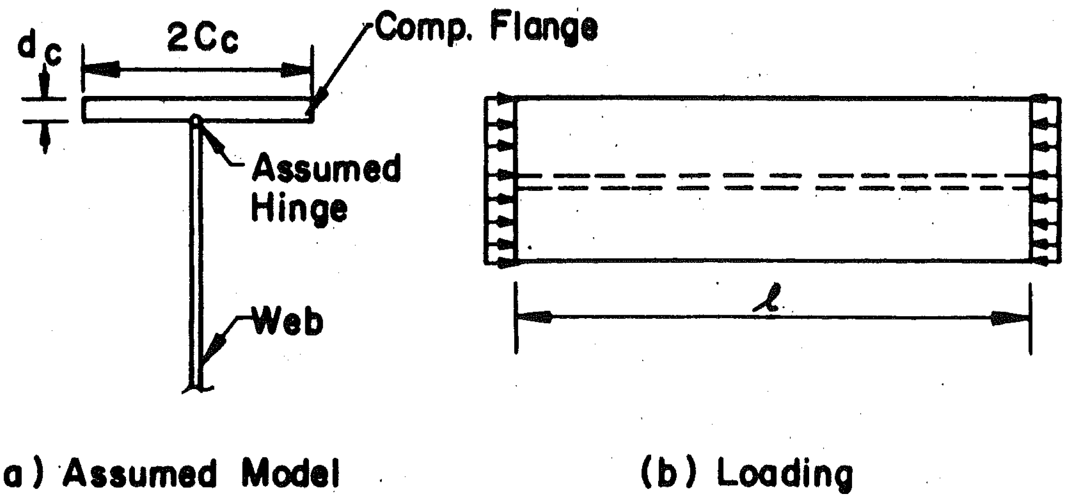

Fig. 9 Torsional Buckling of Compression Flange

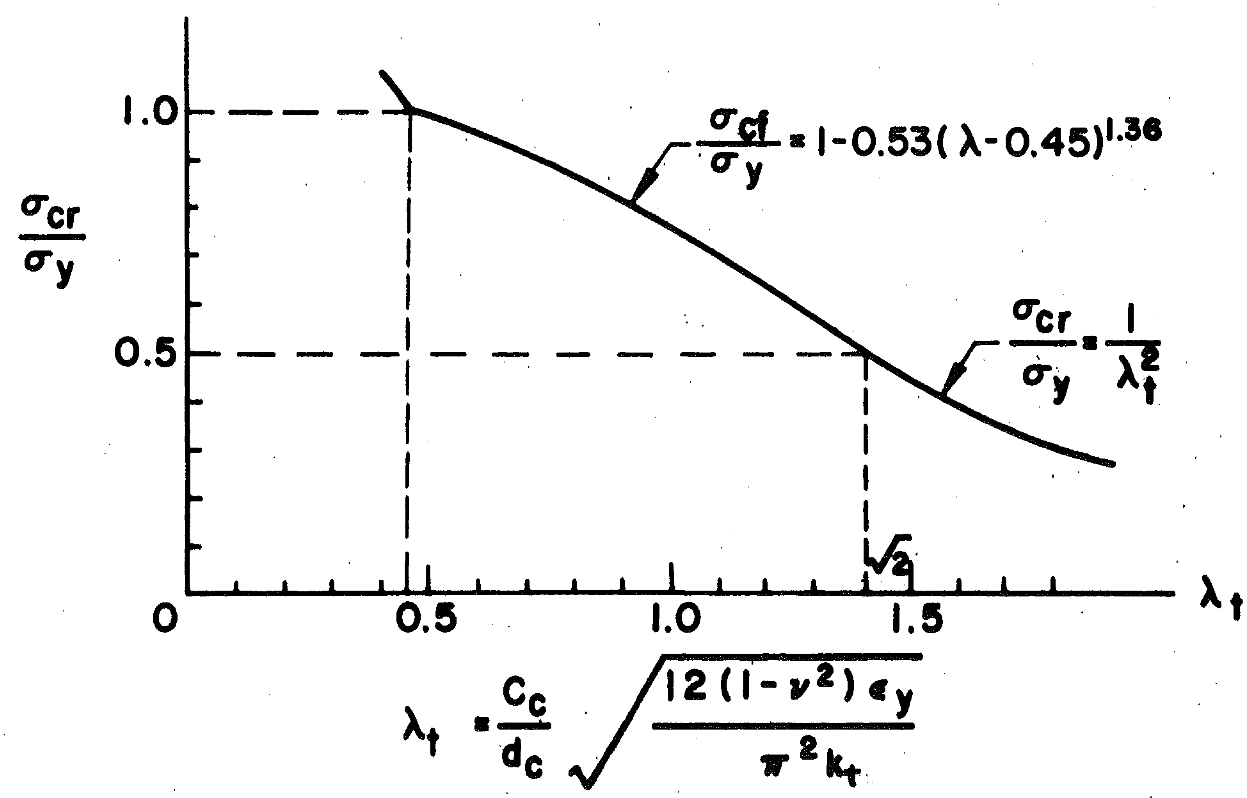

Fig. 10 Torsional Buckling Curve

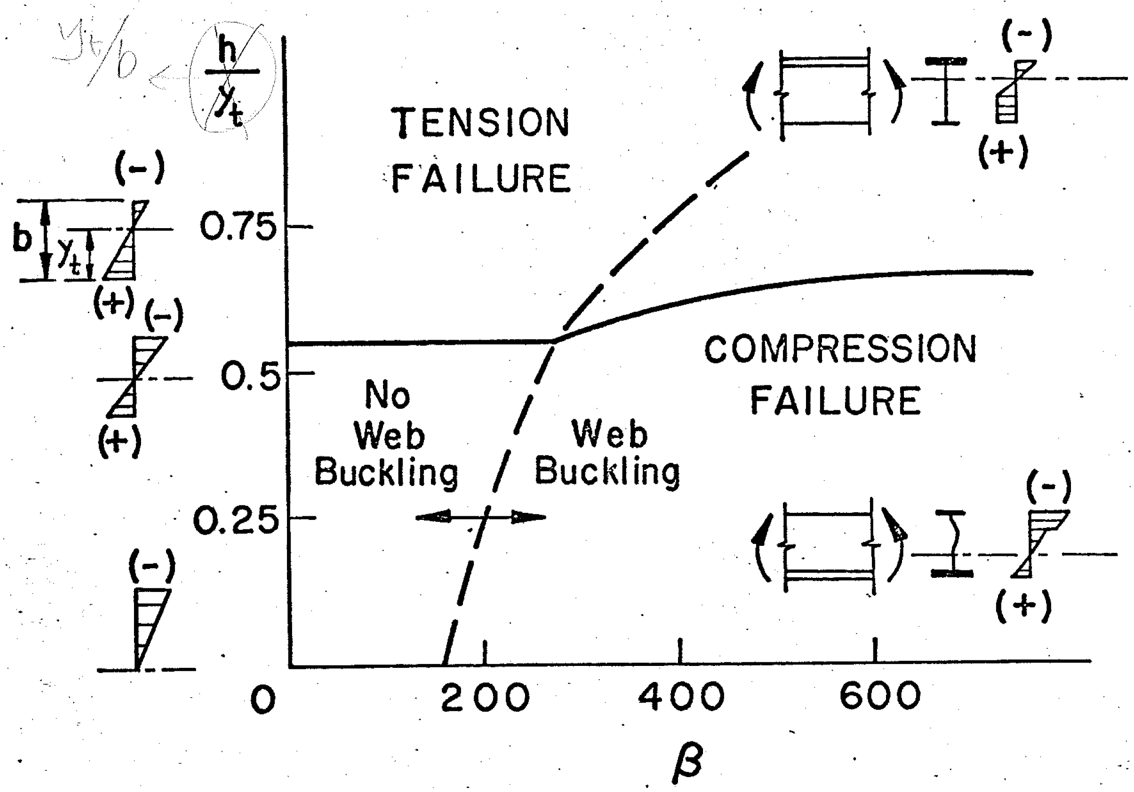

Fig. 11 Modes of Failure

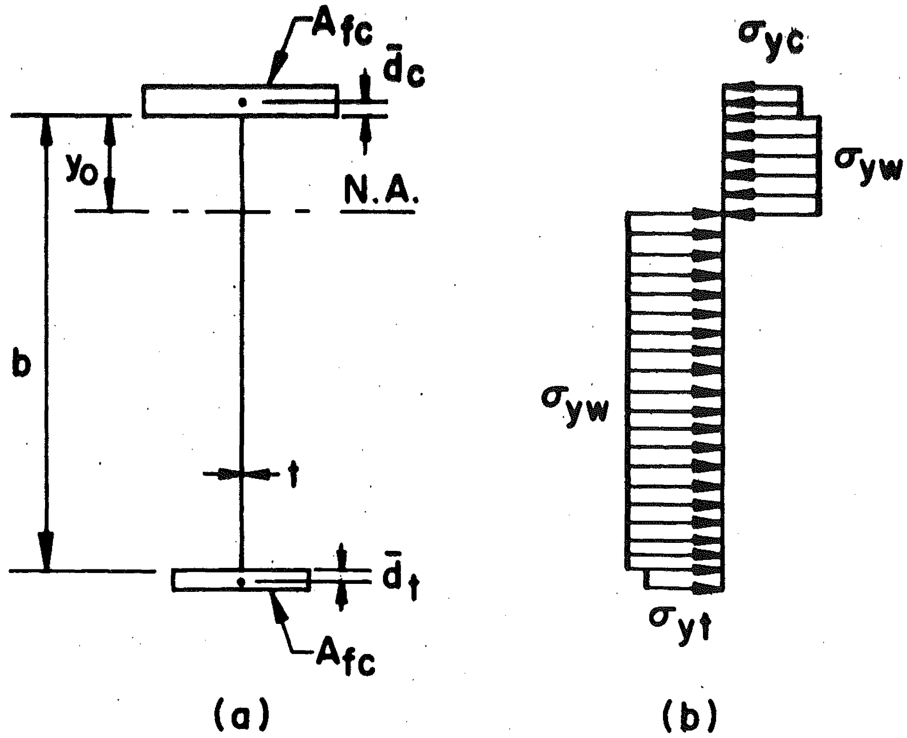

Fig. 12 Plastic Moment - Neutral Axis in the Web

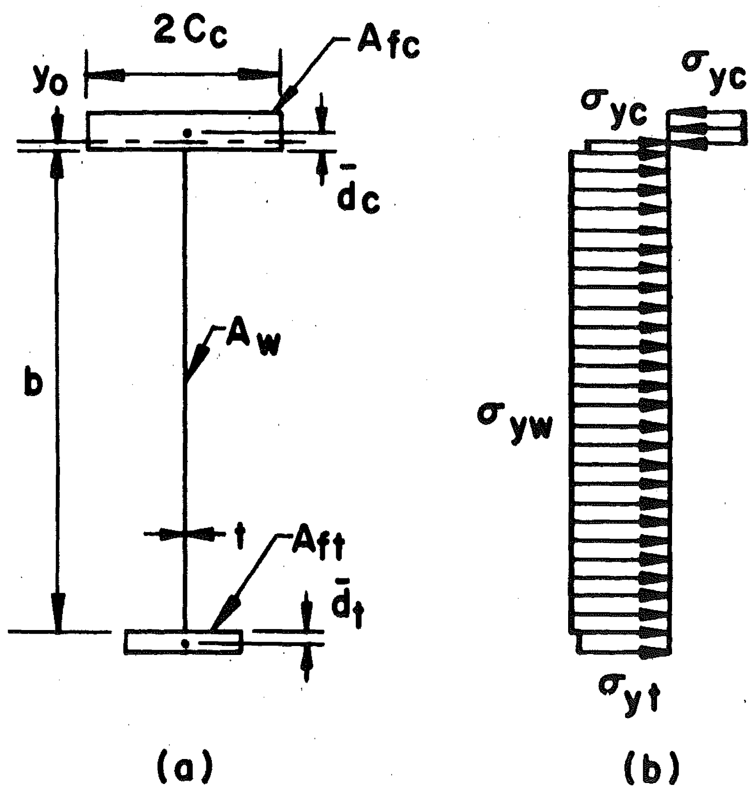

Fig. 13 Plastic Moment - Neutral Axis in the Flange